On the structure of certain K3 surfaces

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On the Structure of certain $K3$ surfaces

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$(\mathrm{I}_4^{*2}) := \{X \to \mathrm{P}_t^1 \mid$ an elliptic $K3$ surface X with a section and two singular fibers both of type I_4^* at $t = 0, \infty \in \mathrm{P}_t^1$ over $\mathrm{C}\}$

E_1, E_2 : elliptic curves

$S = \mathrm{Km}(E_1 \times E_2)$ has an elliptic fibration having a section and 3 singular fibers of type I_4^*, $\mathrm{I}_{\mu_1}^*$, $\mathrm{I}_{\mu_2}^*$ at $\infty, 2, -2 \in \mathrm{P}^1$ $(0 \le \mu_1, \mu_2 \le 1)$.

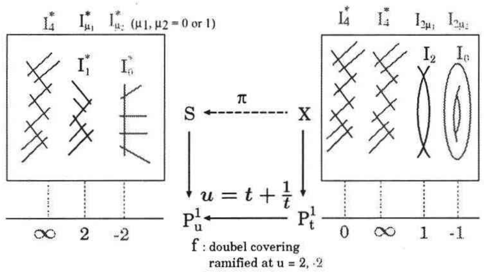
X: the double covering surface of S induced by

$$f : \mathrm{P}_t^1 \to \mathrm{P}_u^1 \,;\, t \mapsto u = t + \frac{1}{t}$$

$$\Downarrow$$

$$X \in (\mathrm{I}_4^{*2})$$

(♣) Conversely is it possible to recover such a Kummer surface S from given $X \in (\mathrm{I}_4^{*2})$?



$$\Downarrow$$

$$X \in (\mathrm{I}_4^{*2})$$

Every $X \in (\mathrm{I}_4^{*2})$ has the following Weierstrass equation

$$y^2 = x^3 - 3t^2(t^4 + a_3t^3 + a_2t^2 + a_1t + 1)x + 2t^3(t^6 + b_5t^5 + b_4t^4 + b_3t^3 + b_2t^2 + b_1t + 1),$$

where

$$b_1 = \frac{3}{2}a_1$$
$$b_2 = \frac{3}{8}{a_1}^2 + \frac{3}{2}a_2$$
$$b_3 = -\frac{1}{16}{a_1}^3 + \frac{3}{4}a_1a_2 + \frac{3}{2}a_3 = -\frac{1}{16}{a_3}^3 + \frac{3}{4}a_2a_3 + \frac{3}{2}a_1$$
$$b_4 = \frac{3}{8}{a_3}^2 + \frac{3}{2}a_2$$
$$b_5 = \frac{3}{2}a_3$$
$${a_1}^4 - 8{a_1}^2a_2 + 32a_1a_3 + 16a_2^2 - 64a_2 - 16{a_3}^2 + 64 \neq 0$$
$${a_3}^4 - 8a_2{a_3}^2 + 32a_1a_3 + 16a_2^2 - 64a_2 - 16{a_1}^2 + 64 \neq 0.$$

$$(\mathrm{I}_4^{*2})_2 := \{X \in (\mathrm{I}_4^{*2}) \mid a_1 = a_3\}$$

$$(\mathrm{I}_4^{*2})_* := \{X \in (\mathrm{I}_4^{*2}) \mid a_1 \neq a_3\}$$

The answer of the question (♣)

$X \in (\mathrm{I}_4^{*2})_2 \Rightarrow$ YES.

$X \in (\mathrm{I}_4^{*2})_*$ and $\rho(X) = 18 \Rightarrow$ NO.

Definition. (Shioda–Inose structure)
A $K3$ surface X admits a Shioda–Inose structure if there exists a symplectic involution ι on X with the rational quotient map $\pi : X \dashrightarrow Y$ of ι such that Y is a Kummer surface and $\pi_* : H^2(X, \mathbf{Z}) \to H^2(Y, \mathbf{Z})$ induces a Hodge isometry $T_X(2) \simeq T_Y$.

If X admits a Shioda–Inose structure, the complex torus such that $Y = \mathrm{Km}(Z)$ gives a diagram

$$X \dashrightarrow Y \dashleftarrow Z$$

of rational maps of degree 2. This diagram induces a Hodge isometry

$$T_X \simeq T_Z.$$

Theorem 1.

If $X \in (\mathrm{I}_4^{*2})_2$, then X admits a Shioda–Inose structure.

Theorem 2.

If $X \in (\mathrm{I}_4^{*2})_*$ and Picard number of X is 18, then X does not admit a Shioda–Inose structure. However X is isomorphic to a Kummer surface of product type.